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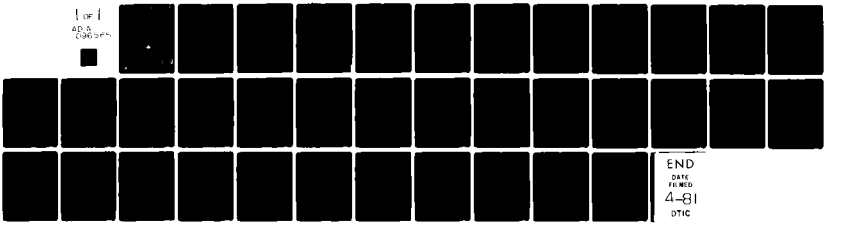

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(14) USAFA-TR-81-2

# (6) AN APPLICATION OF MARGINAL RETURN ANALYSIS TO EFFICIENT REORDER POINT CALCULATION

(10) ROBERT A. RAPPOLD, LT COLONEL, USAF

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(9) FINAL REPORT

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| REPORT DOCUMENTATION PAGE | READ INSTRUCTIONS BEFORE COMPLETING FORM |
|---|---|
| 1. REPORT NUMBER: USAFA TR 81-2 / 2. GOVT ACCESSION NO.: AD-A096 565 | 3. RECIPIENT'S CATALOG NUMBER: |
| 4. TITLE *(and Subtitle)*: An Application of Marginal Return Analysis to Efficient Reorder Point Calculation | 5. TYPE OF REPORT & PERIOD COVERED: Final Report |
| | 6. PERFORMING ORG. REPORT NUMBER: |
| 7. AUTHOR(s): Robert A. Rappold, Lt Col, USAF | 8. CONTRACT OR GRANT NUMBER(s): |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS: Department of Mathematical Sciences, US Air Force Academy, CO 80840 | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: |
| 11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Logistics Management Center, Gunter AFS, AL | 12. REPORT DATE: October 1980 |
| | 13. NUMBER OF PAGES: 33 |
| 14. MONITORING AGENCY NAME & ADDRESS*(if different from Controlling Office)*: | 15. SECURITY CLASS. *(of this report)*: |
| | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE: |

16. DISTRIBUTION STATEMENT *(of this Report)*

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT *(of the abstract entered in Block 20, if different from Report)*

18. SUPPLEMENTARY NOTES

19. KEY WORDS *(Continue on reverse side if necessary and identify by block number)*

Logistics, Reorder Point, Order Quantity, Marginal Return, Constant-Poisson distribution.

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

One of the important uses of an EOQ item's probability distribution of demand during leadtime is to establish the reorder point. The current Air Force System computes the reorder point of each item independently. The research presented here shows that a more efficient calculation considers a homogeneous grouping of parts as an entity and bases the reorder point on the individual item's marginal return. In other words, the reorder point for each item is based on the gain in safety stock insurance per dollar invested relative to other items in the homogeneous grouping.

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Data from six federal stock groups and three Air Force bases are analyzed. Approximately 3,000 items are involved in the performance comparisons. An examination is made of modifying the marginal return to accommodate the "essentiality" of individual items. Following this effort the reorder point for each item is then coupled with a more nearly optimal order quantity.

# AN APPLICATION OF MARGINAL RETURN ANALYSIS TO EFFICIENT REORDER POINT CALCULATION

by

Lt Col R. A. Rappold
Dept of Mathematical Sciences
USAF Academy, CO

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February 1981

# PREFACE

The research reported here was accomplished for the Air Force Logistics Management Center at Gunter AFS, AL. Of primary interest is the efficient calculation of the reorder point for EOQ items. Coupled with this is a stockage policy for base level allocation of supply funds. This research also contributes to the basic understanding of the interplay between order quantity and reorder point for EOQ items.

## SUMMARY

One of the important uses of an EOQ item's probability distribution of demand during leadtime is to establish the reorder point. The current Air Force system computes the reorder point of each item independently. The research presented here shows that a more efficient reorder point can be established by considering a homogeneous grouping of parts as an entity and examining the marginal return of each item in the group. The marginal return scheme is based on the gain in safety stock insurance per dollar invested. Thus, the item which maximizes the marginal return will have its reorder point increased while the other items' reorder points remain the same. This scheme can be iterated until either a performance level is reached or a fund level is reached. At the conclusion of the iteration each item in the grouping will have its reorder point established.

The marginal return scheme may be started at any initial reorder point level desired. This research examines two applications. In the first, the reorder point is initially set to zero for each item and the marginal return allocation begins. In the second, each item has its reorder point set to accommodate the expected number of demands during leadtime and then the marginal return scheme is applied. The results show that there is almost no difference in the system performance, as measured in this research, in the two cases.

The marginal return scheme used in this research tends to favor lower cost items. In reality, an item's criticality should also be considered when purchasing safety stock. To accommodate this necessary feature, each item's marginal return is modified by an "essentiality" factor. The resulting reorder point calculations again show very little difference either in system performance or costs.

A complete policy for an inventory system should include both the reorder point and order quantity. Thus, at this point the research turns to developing a full policy. The reorder point is again calculated by the modified marginal return and in addition the optimal order quantity commensurate with the reorder point is calculated for each item. The resulting full policy is then evaluated in terms of the total variable cost.

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## SECTION 1

## BACKGROUND AND INTRODUCTION

The research reported here is in support of an implementation plan for DOD Instruction 4140.45 [1] (DODI). This instruction pertains to a standard stockage policy for consumable secondary items at the so-called retail level of inventory. Implied in the DODI is the determination of a methodology for calculating the reorder point and order quantity for individual EOQ items. In the presence of continually decreasing stock funds, it becomes imparative to allocate the available funds in the most efficient manner possible.

Mitchell, et. al., [4] in support of the DODI have shown that the Constant-Poisson Model adequately approximates the demand process during typical lead-times experienced by Air Force EOQ items. This model assumes that customer arrivals are described by a Poisson process and each customer requests a constant lost size. The customer arrival rate parameter and the lot size are estimated for each item from Air Force historical data. Additionally, it has been shown [4] that the reorder points established by this model are relatively insensitive to typical leadtimes experienced by the Air Force. As a consequence, the Constant-Poisson Model and a constant leadtime will be assumed throughout this report.

Problems concerned with the allocation of resources (in this case stock monies) can be handled very nicely by an incremental analysis [2]. First, it is relatively easy to employ at each step, and second, it can be terminated by a variety of measures of merit. The marginal return scheme developed in this report maximizes the incremental probability of satisfying customer demands during leadtime per additional dollar expended. For this research, the scheme is defined by:

$$MR = \frac{\text{Probability (next customer arrival during leadtime)}}{\text{Lot Size X Unit Cost}}$$

This scheme is applied iteratively and units are added to the reorder point of the item which has the largest marginal return.

The primary objective of this research is to apply a marginal return analysis to the reorder point problem and to demonstrate the adequacy of the approach. A secondary objective is to couple the calculated reorder points with the order quantity and thus develop the operational policy.

The remainder of this paper is organized as follows: Section 2 covers the application of the marginal return methodology and Section 3 contains the recommendations. In addition, there are three appendices which present the following: Appendix A - formulation of the marginal return scheme, Appendix B - formulation of the total variable cost equation, and Appendix C - presentation of the results for various stock accounts.

## SECTION 2

## APPLICATION OF MARGINAL RETURN

In dealing with problems concerned with the allocation of resources, an incremental analysis which somehow maximizes the return for each additional dollar expended is very appealing. In an environment of limited funds, existing today for normal stock replenishment, an incremental analysis and the resulting policy formulation is almost a necessity. The marginal return scheme developed and applied in this report is such an incremental analysis. As applied here, the scheme incrementally maximizes the safety stock during stock replenishment leadtime per dollar invested. This is accomplished by iteratively searching a set of items and increasing the reorder point for the single item which maximizes the return. Appendix A gives the formulation of the marginal return scheme used in this report.

### Data

For studies of this nature several Air Force bases have been providing demand data to the Air Force Logistics Management Center (AFLMC). The data used to test the marginal return scheme are from three of these bases. Table 1 shows the bases and federal stock groups which comprised the data set.

TABLE 1. Data

| Base | Federal Stock Group | # Items |
|---|---|---|
| Dover | 15 - Aircraft Structures | 500 |
| | 59 - Electronics | 1491 |
| | 66 - Flight Instruments | 249 |
| Minot | 29 - Engine, Miscellaneous | 257 |
| | 31 - Bearings | 174 |
| Randolph | 53 - Screws, Nuts, Bolts | 294 |

The description of a stock group is not all encompassing, but is representative of items in the group

Marginal Return With Zero Reorder Point

The Air Force currently calculates the reorder point on a by-item basis. The calculation is based on the cumulative probability of satisfying a prescribed percentage of demand during stock replenishment leadtime. While the calculation is simple, it ignores the questions of total fund levels and the relative value of different items. Both of these considerations can be included in the marginal return approach and they will be addressed as a stockage policy is developed. The scheme used in this research considers a homogeneous grouping, in this case a federal stock group, as a single entity. The reorder point of the item within the group that maximizes the marginal return at each iteration is increased. Since a stock group is considered an entity, the leadtime stockout probabilities are averaged over the entire group.

Figure 1 shows the reorder point costs (reorder point units multiplied by the unit cost) as a function of the probability of stockout during leadtime for the Dover 59 federal stock group. The by-item curve represents the current

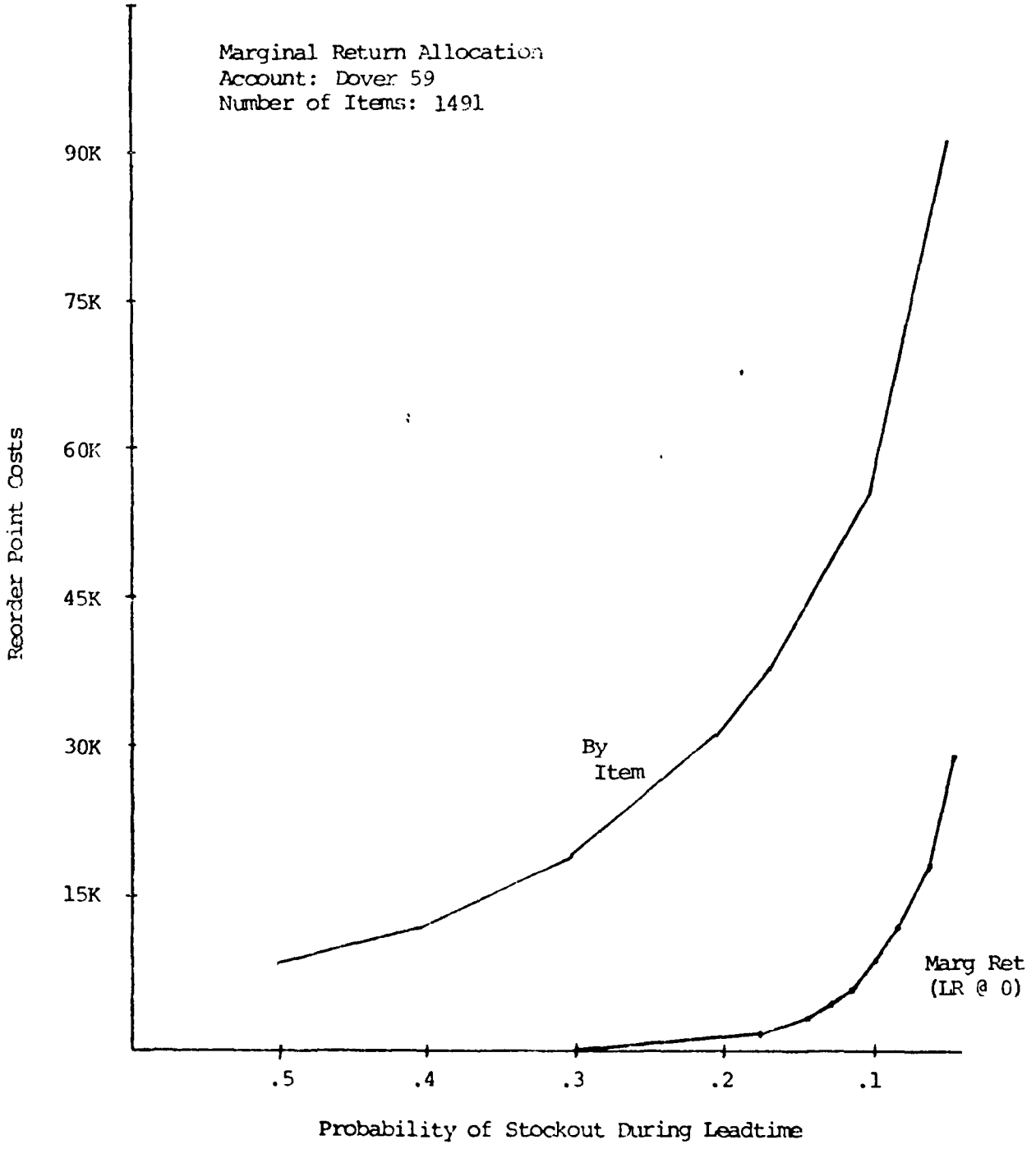

Marginal Return Allocation
Account: Dover 59
Number of Items: 1491
90K
75K
60K
45K
30K
15K
Reorder Point Costs
By
Item
Marg Ret
(LR @ 0)
.5
.4
.3
.2
.1
Probability of Stockout During Leadtime

Figure 1

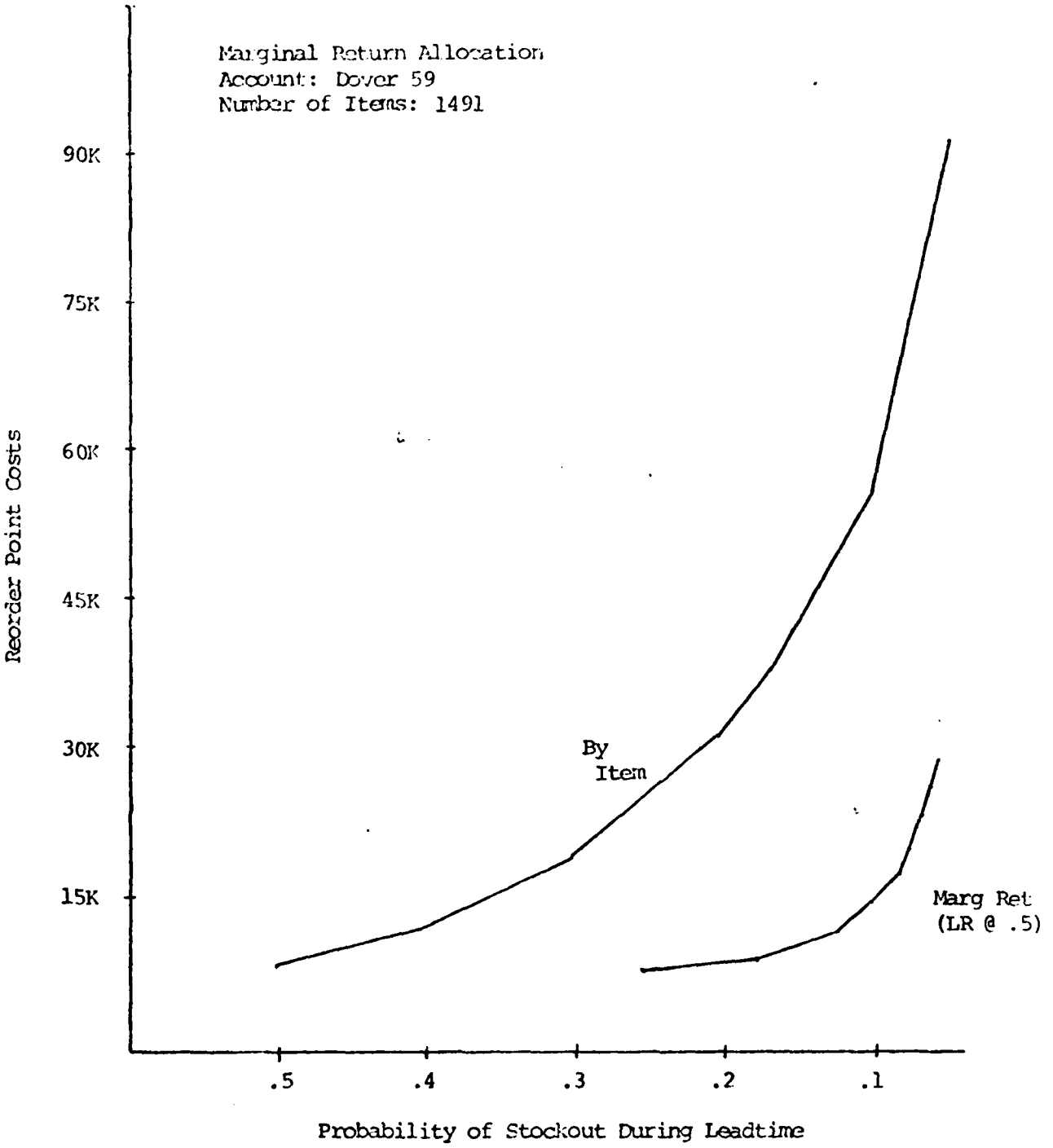
approach of forcing each item to meet the desired probability. The marginal return curve shows the results of initially setting each item's reorder point to zero and then employing the marginal return scheme to selectively increase the reorder point of items within the group. The probabilities reflected on this curve are an average for the stock group. The figure shows a wide range of stockout probabilities but both analyses could have been terminated by an upper limit on the total reorder point costs as well as a lower limit on the probability of stockout during leadtime.

A potential problem with the marginal return scheme is the individual stockout probabilities for the items within the group. Although the average stockout probability may be very low for the group as a whole, some items may have an unacceptably high stockout probability.

Marginal Return With Expected Demand

There are several approaches to ensure the potential problem mentioned above does not occur. The approach used here not only solves the problem computationally but should be more palatable to supply people in the field. This modified approach initially sets the reorder point for each item as close as possible to the expected demand during leadtime and then adds additional leadtime protection by the marginal return scheme. Since the leadtime demand process is assumed to be the Constant-Poisson, it is not possible to achieve exactly a .5 probability of stockout for every item. Thus, the cumulative probability that is closest to .5 determines the initial reorder point.

Figure 2 shows the results of this modified marginal return scheme and the current by-item approach. Since the reorder points are initially set as close as possible to accommodate the expected demand during leadtime, the marginal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.4
.3
.2
.1
Probability of Stockout During Leadtime

Figure 2

return curve represents the leadtime safety stock investment. Again the curves could have been terminated by either an upper limit on reorder point costs or lower limit on stockout probability.

The marginal return scheme employed for the curve on Figure 2 alleviates the potential problem previously mentioned. In addition, it should be more acceptable to personnel in the field since the reorder point policy would cover the expected demand for each item. However, this scheme does not solve the main drawback of a marginal return allocation. The next subsection covers this problem and its solution.

Marginal Return With Essentiality

The main limitation of the marginal return scheme used here is the prejudice towards low cost items. For example, if two items had the same demand patterns but one cost twice as much as the other, then the cheaper item would have a marginal return twice as large as the more expensive item for the same probability level. With this in mind, it is not difficult to see that the average probability of a group may be comprised of high reorder points for low cost items and low reorder points for high cost items. To be useful in an operational environment the marginal return approach must be amenable to the criticality of different items. We must recognize that there are parts which are mission essential and should be stocked at high levels, while there are others which could be stocked at lower levels. The Air Force currently attempts to account for the criticality of items by using a "Stockage Priority Code" when determining reorder points and order quantities.

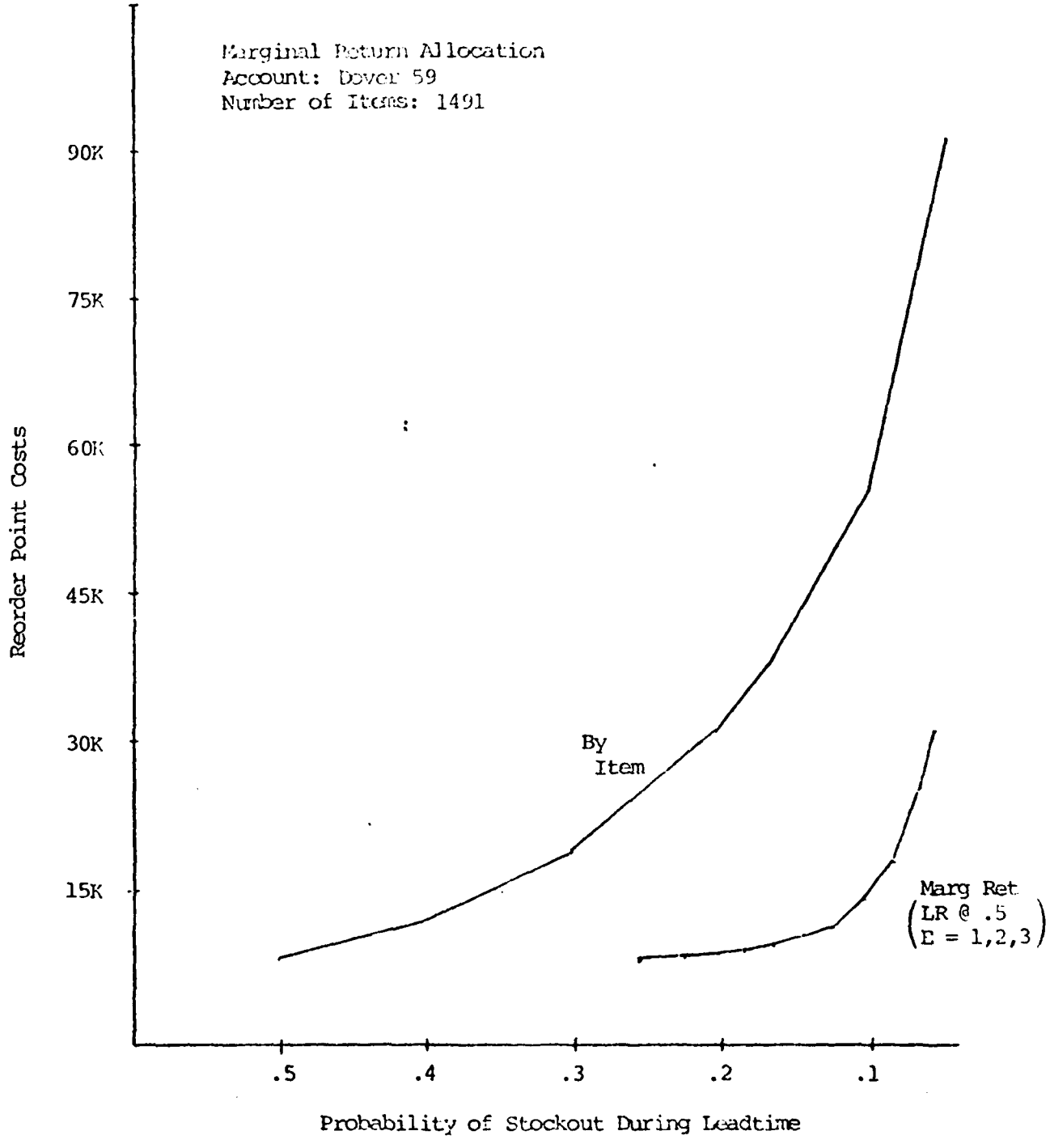
The marginal return scheme must be modified to account for the "essentiality" of items. The method used here is an example of what can be done to modify the scheme. First we will define an essentiality factor with values of 1, 2, or 3. The marginal return values will be modified with the essentiality as a multiplicative factor; that is, the marginal return for each item will now be given by:

$$MR = \frac{\text{Essentiality X Probability (Next Customer Arrival During Leadtime)}}{\text{Lot Size X Unit Cost}}$$

In this report the essentiality factor was assigned randomly to the items in the account. Additionally, the distribution was uniform; thus, approximately one-third of the items had an essentiality factor of one, two, and three.

Figure 3 shows the results of the current by-item approach with the essentiality modified marginal return scheme with all reorder points initially set to accommodate the expected demand during leadtime. This modified marginal return scheme appears to offer a workable compromise between optimality and application in the field. Initially all reorder points are established to accommodate, as close as possible, the expected demand during leadtime. Secondly, the scheme demonstrates a technique for coping with the criticality of the item in accomplishing the mission. Thirdly, additional safety stock is purchased by incrementally maximizing the return per additional dollar expended.

To establish a complete inventory policy two variables must be specified for each item: 1) when to order (reorder point), and 2) how much to order (order quantity). Up to this point the research has been concerned only with determining the reorder point. With the essentiality modified marginal return established, we will now pursue the development of a complete inventory policy.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By Item
Marg Ret
(LR @ .5
Σ = 1,2,3)
.5
.4
.3
.2
.1
Probability of Stockout During Leadtime

Figure 3

Order Quantity Determination

Appendix B gives the notation and develops the total variable cost expression applicable to the Constant-Poisson Model. Using this cost model there are several possible ways to calculate the order quantity. One of the more standard ways is to use the well-known Wilson-Q:

$$Q = \sqrt{\frac{2\ LDA}{IC}} \qquad (1)$$

This expression for Q is easy to calculate but it is a poor choice to use in conjunction with the marginal return approach, as will be shown below. At the other extreme of complexity is the joint reorder point-order quantity optimal. However, this method ignores the reorder point established by the marginal return allocation. A compromise approach will now be developed. Assuming for a moment that Q is a continuous random variable, it is possible to differentiate the total variable cost expression with respect to Q, set equal to zero, and obtain:

$$Q = \sqrt{\frac{2LD[A+\pi L(\lambda t - R + \sum_{x=0}^{R-1} (R-x)f(x;\lambda t))]}{IC}} \qquad (2)$$

We now propose to mesh this expression for Q with the reorder point established by the marginal return. The complete inventory policy would iterate the essentiality modified marginal return to calculate the reorder points for each item in a group and then calculate the order quantity for each item by Equation 2 and round the result.

The Air Force is currently using an expression similar to the Wilson-Q formula for calculating the order quantities. However, this expression ignores the backorder penalty cost which is included in Equation 2. If a marginal return

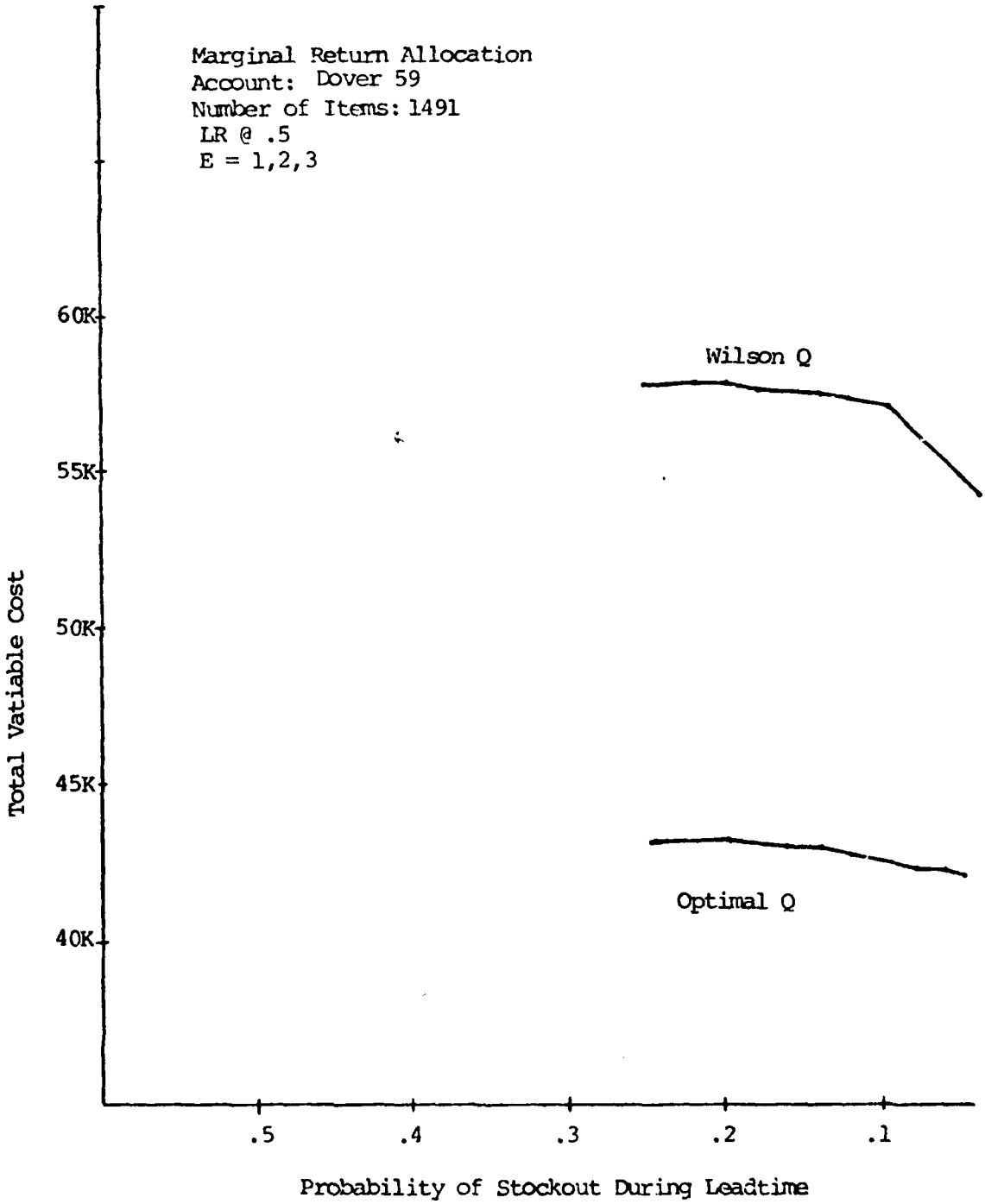
is used to establish the reorder points, then the probability of a backorder may vary widely from item to item in a given group. It would seem prudent to use Equation 2 to determine the order quantity for each item and account for the probability of a stockout.

Figure 4 shows the effect on the total variable cost by using the Wilson-Q versus the "optimal" Q determined from Equation 2 for calculating the order quantities. For both curves the reorder points are initially set to accommodate the expected leadtime demand and then are iteratively increased by the essentiality modified marginal return. The curves demonstrate quite dramatically the effect of including the backorder penalty offset in the calculation of the order quantity.

Summary

Figure 1 was used to demonstrate the usefulness of a marginal return allocation of reorder point money. Although the curve does show the utility of a marginal return approach, this basic approach has some limitations in an operational environment. Figure 2 shows the results of an operational fix whereby every item initially has its reorder point set as close as possible to the expected leadtime demand and the marginal return allocation is started at this point. Probably the biggest drawback to the marginal return scheme used up to this point is the favoring of low cost items. To be implemented in the field the approach must be able to account for the different criticalities of the items in a group. Figure 3 shows the results of attempting to resolve this problem by using essentiality as a multiplicative factor in defining the marginal return.

To develop a complete inventory policy a procedure must be given for calculating the reorder point and order quantity for every item. The essentiality modified marginal return appears to efficiently determine the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50K
45K
40K
Total Vatiable Cost
Wilson Q
Optimal Q
.5
.4
.3
.2
.1
Probability of Stockout During Leadtime

Figure 4

reorder points. To take full advantage of this scheme in a complete policy, the order quantity calculation should be consistent with the established reorder point. Equation 2 takes into account the interdependence of the reorder point and order quantity. Figure 4 shows the results of the "optimal" Q and Wilson-Q on the total variable cost.

Appendix C gives the same sequence of figures for the other stock accounts considered in this research.

# SECTION 3

# RECOMMENDATIONS

The results presented in this report appear very promising for Air Force adoption. However, it would be misleading to attempt far reaching conclusions from the results of six data sets. For this reason the AFLMC should become familiar with the procedure and apply it to other data sets.

The essentiality modified marginal return was devised to illustrate the effectiveness of the general approach. The Air Force currently has a "Stockage Priority Code", which is an attempt to quantify the relative importance of various items. This factor could easily be incorporated as the basis of essentiality and should be included in further verification efforts.

As a last remark, I recommend that a full base account (all federal stock groups) be used in comparative efforts between the marginal return approach and the current system. Coupled with the widely different groups studied here, the results for an entire base would form an in-depth package for evaluation.

## APPENDIX A

## MARGINAL RETURN FORMULATION

The marginal return scheme to be developed below has certain intuitively appealing features. In the face of decreasing stock funds, an approach which maximizes the incremental return per additional dollar expended is very attractive. The marginal return algorithm is an attempt to accomplish this by incrementally adding to the safety stock during leadtime (by setting the reorder point) for a given set of items.

For a given set of n items it will be assumed that the Constant-Poisson model describes the process of customer arrival and units per demand. Further, for each item in the set the daily customer arrival rate, $\lambda_i$, and the units per demand lot size, $L_i$, are assumed known. With the additional assumption of a known constant leadtime, t, all of the parameters are specified.

The marginal return (MR) for each item is defined by the expression:

$$MR = \frac{\text{Probability (Next customer arrival during leadtime)}}{\text{Lot Size x Unit Cost}} . \quad \text{(A-1)}$$

The marginal return scheme can be started at any initial reorder point level desired. For example, if the reorder point is initially set to zero for each item in the set, then the marginal return for each item is simply the probability of the first customer arriving during leadtime normalized by the cost to accommodate that customer. On the other hand, if a particular item's reorder point is set to accommodate three customer arrivals, then the current marginal return for this item is the probability of the fourth customer arriving during leadtime again normalized by the cost to satisfy the demand.

In algorithmic form the marginal return process is:

Step 1: Calculate $MR_i$, $1 \leq i \leq n$

Step 2: Find J such that $MR_J = \max_{1 \leq i \leq n} \{MR_i\}$

Step 3: Add $L_J$ units to the reorder point for item J

Step 4: Update marginal return for item J

Step 5: Check termination criteria - stop if satisfied

Step 6: Go to Step 2

This process is designed to incrementally allocate reorder point money in the most advantageous manner. The algorithm above is based on probability per dollar and at each iteration the reorder point is increased for the item which maximizes this quantity. An item's reorder point determines the safety stock during a stock replenishment leadtime; thus, the marginal return process may also be viewed as maximizing the safety stock per dollars expended.

## APPENDIX B

## TOTAL VARIABLE COST FORMULATION

Mitchell, et.al. [4] have shown that the Constant-Poisson Model adequately describes the customer arrival and demand patterns during leadtime experienced by Air Force EOQ items. This model will be used in the development of the total variable cost model used in this report. The following notation will be used:

Q = order quantity (units)

L = customer lot size (units/customer)

D = expected number of customers annually (customers/year)

A = order cost ($/order)

I = inventory holding cost ($/unit/year)

C = unit cost of an item (R/unit)

R = number of customers included in the reorder point (customers)

$\pi$ = backorder penalty cost ($/unit)

t = stock replenishment leadtime (constant in days)

$\lambda$ = customer arrival rate (customers/day)

$f(x;\mu)$ = Poisson frequency function with parameter $\mu$

LR = reorder point (units)

The values for A and I used in the research are the currently accepted values used by the AFLMC. Additionally, the value used for $\pi$ is the implied value for a given probability level of satisfying demand during leadtime.

For a particular stock item the total variable cost (TVC) per year can be expressed as

$$TVC = OC + HC + BC \qquad (B\text{-}1)$$

where:

OC = total annual order costs

HC = total annual holding costs

BC = total annual backorder costs

The order costs can be easily calculated as

$$OC = \frac{LDA}{Q} \tag{B-2}$$

Similarly, the holding cost is expressed as

$$HC = (\frac{Q}{2} + LR - \lambda tL)IC \tag{B-3}$$

This formulation of the holding costs assumes that the expected number of backorders is negligible with respect to the magnitude of the other quantities and calculates the expected on hand inventory as the net inventory. It should be noted that the expressions for the order and holding costs are consistent with the "typical" inventory model; see e.g. Hadley and Whitin [3].

The third cost term, total annual backorder costs, can be determined by a variety of methods. Since the probability function of customer demand during leadtime is known (Constant-Poisson) it is straightforward to calculate the backorder order costs as the product of the penalty cost, expected cycles per year, and the expected number of units short per cycle. Thus,

$$BC = \pi(\frac{LD}{Q})(L\sum_{x=R}^{\infty}(x-R)f(x;\lambda t) \tag{B-4}$$

Substituting equations B-2, B-3, and B-4 into B-1 yields the expression for the total variable cost per year:

$$TVC = \frac{LDA}{Q} + (\frac{Q}{2}+LR-\lambda tL)IC + \frac{\pi L^2 D}{Q}\sum_{x=R}^{\infty}(x-R)f(x;\lambda t). \tag{B-5}$$

# APPENDIX C

This appendix gives a sequence of graphs paralleling the development given in Section 2. All of the data sets analyzed in this research are reflected on these graphs. Each data set, except RANDOLPH 53, contained items with the entire spectrum of unit costs while RANDOLPH 53 contained only those items that cost more than five dollars per unit.

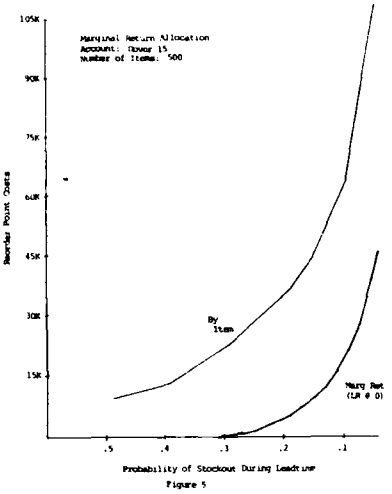
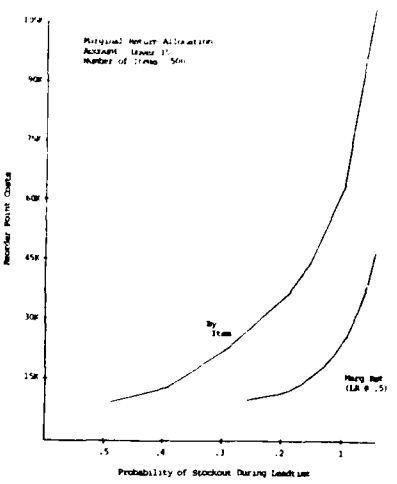



Figure 5

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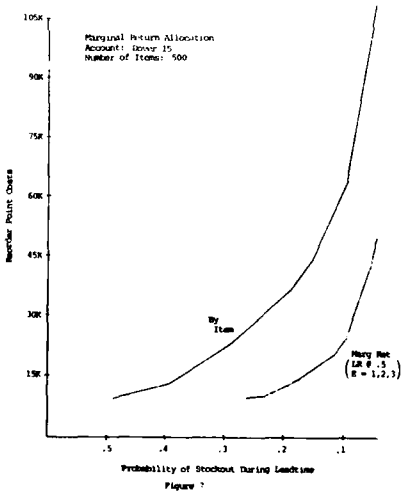



Figure 7

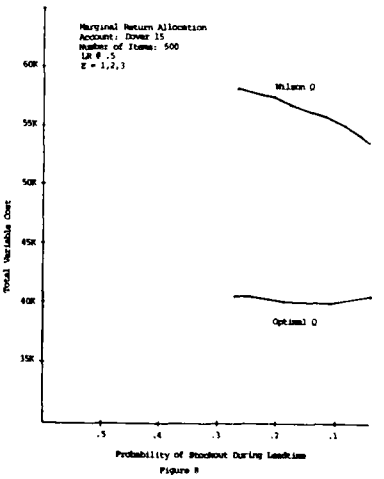



Figure 8

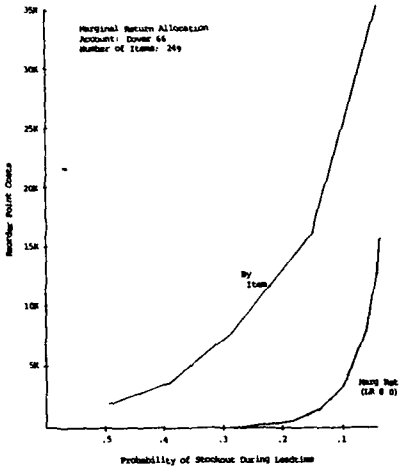


Figure 9

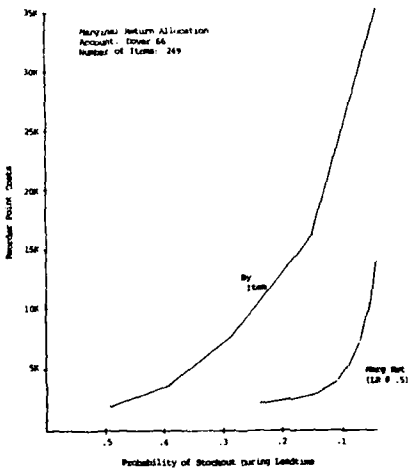


Figure 10

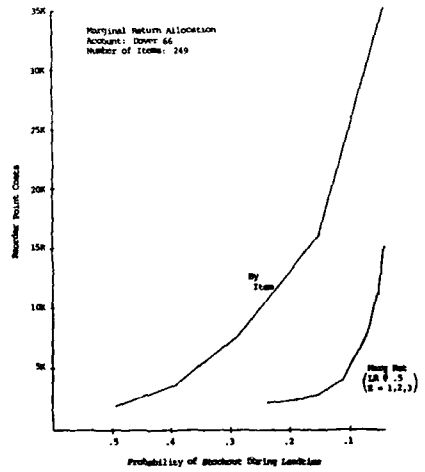


Figure 11

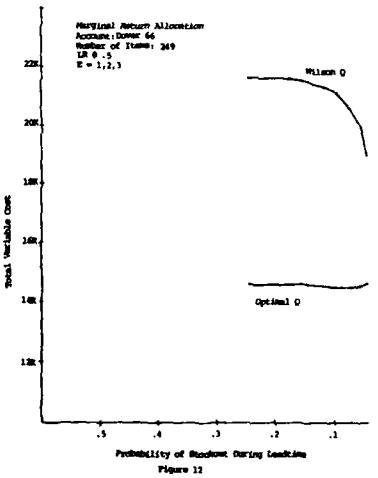


Figure 12

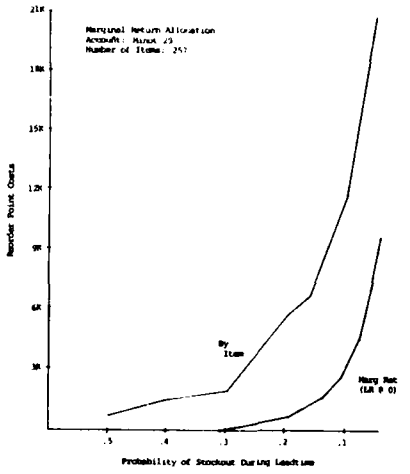


Figure 13

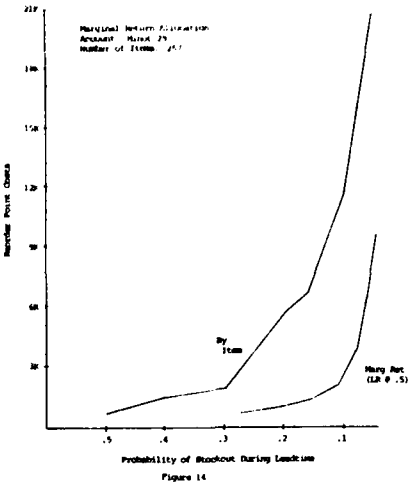


Figure 14

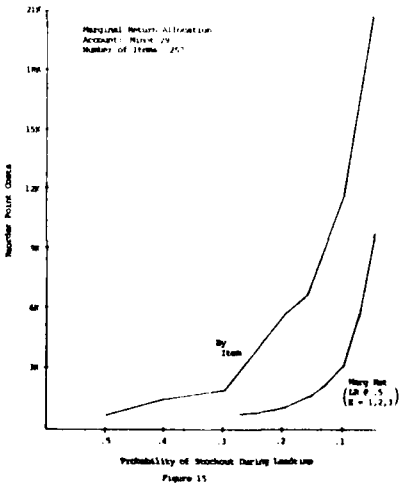


Figure 15

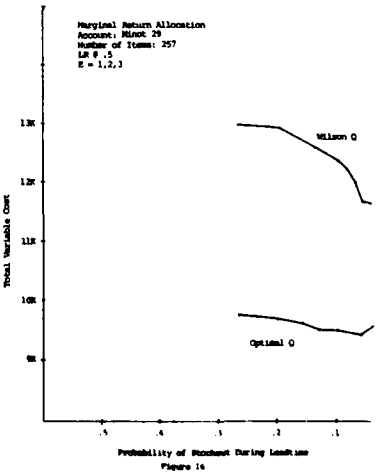
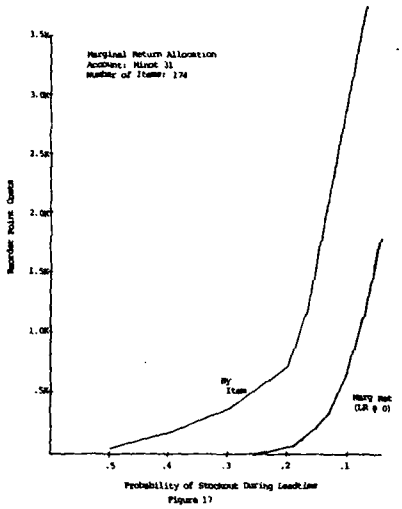


Figure 16



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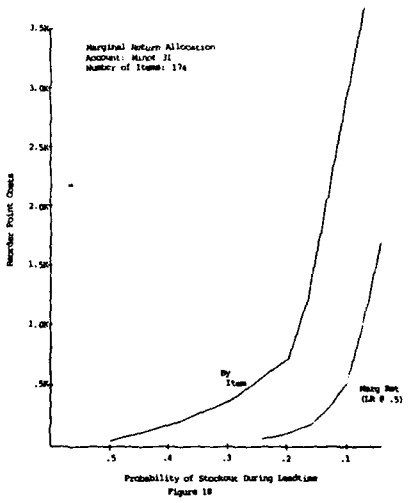

Figure 18

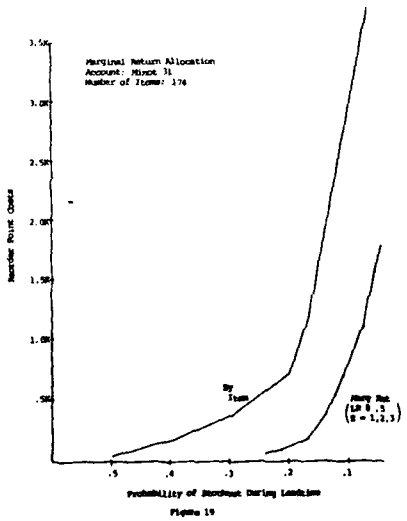

Figure 19

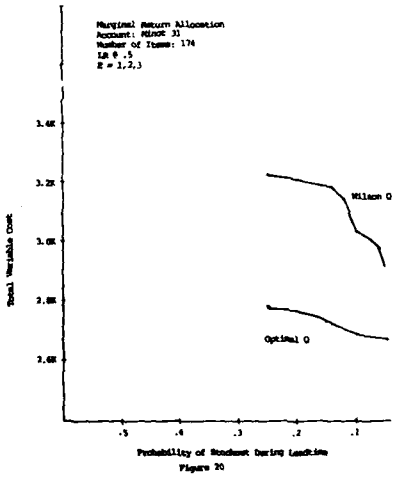

Figure 20

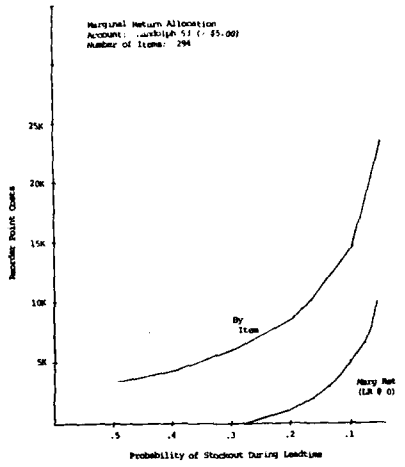


Figure 21

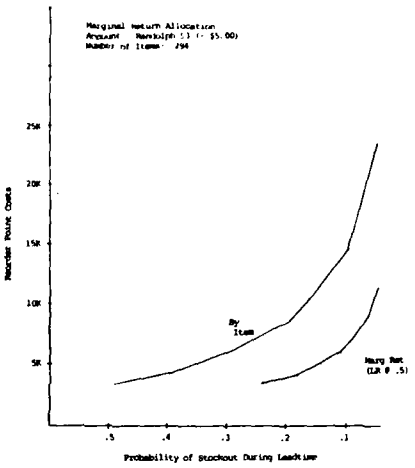


Figure 22

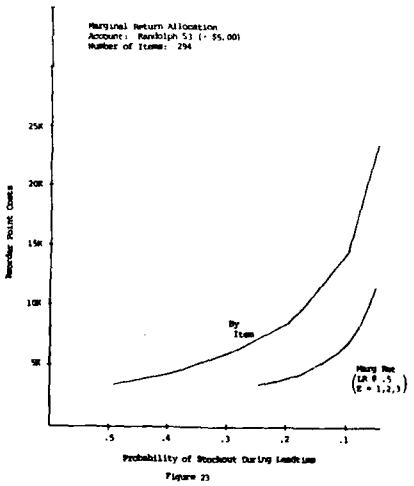


Figure 23

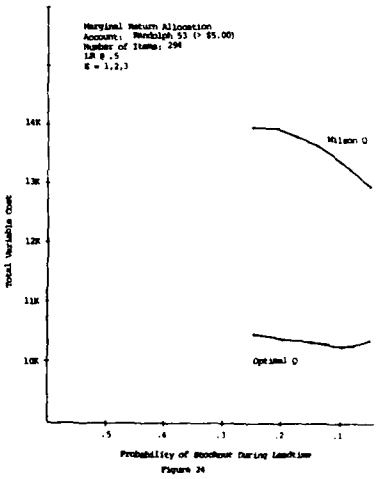


Figure 24

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3. Hadley, G. and T. M. Whitin, Analysis of Inventory Systems, Prentice-Hall, Inc., Englewood Cliffs, New York, 1963.

4. Mitchell, C. R., R. A. Rappold, and W. B. Faulkner, "An Analysis of Air Force Economic Order Quantity Type Inventory Data with an Application to Reorder Point Calculations, USAFA TR-80-6, USAF Academy, 1980.